

$$x = 0.999\bar{9}$$

is  $x < 1$  ?

$$\pi = 3 + \frac{1}{10} + \frac{4}{100} + \frac{1}{1000} + \frac{5}{10000} + \dots$$

2 major ideas:

- factor and cancel
- multiply top & bottom by conjugate  
[[rationalize the numerator]]

Purpose: cancel 0s on top and bottom  
[[or cancel infinities on top and bottom]]

Note: can be used any time you can factor!

# IMPORTANT

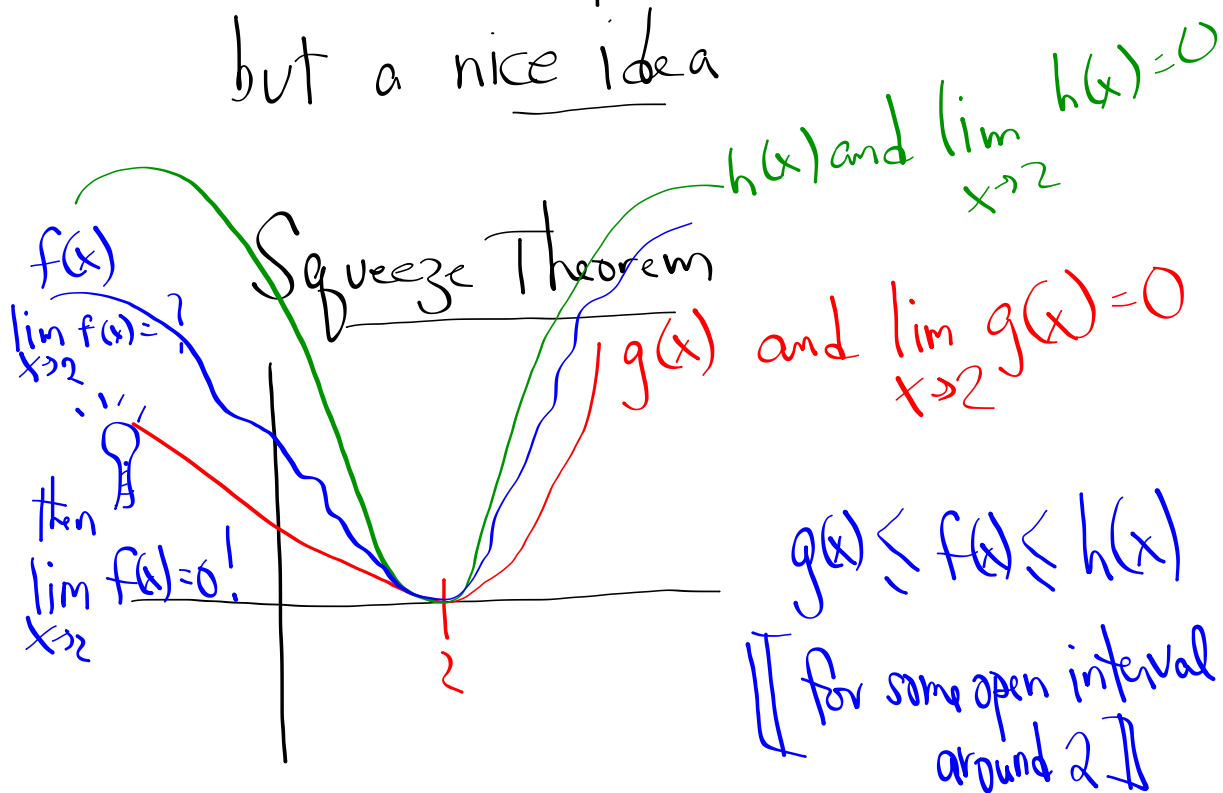
$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

Beware of  
mindless  
pattern  
matching!

Cancelling  
 $x$ s  
in  $\frac{\sin x}{x}$  is  
A SIN!

less important ...  
but a nice idea



Global  
everywhere

Local  
"near a place"  
or point

globally bounded function

$$f(x) = -2 \leq \sin x \leq g(x) = +2$$

$$\lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0?$$

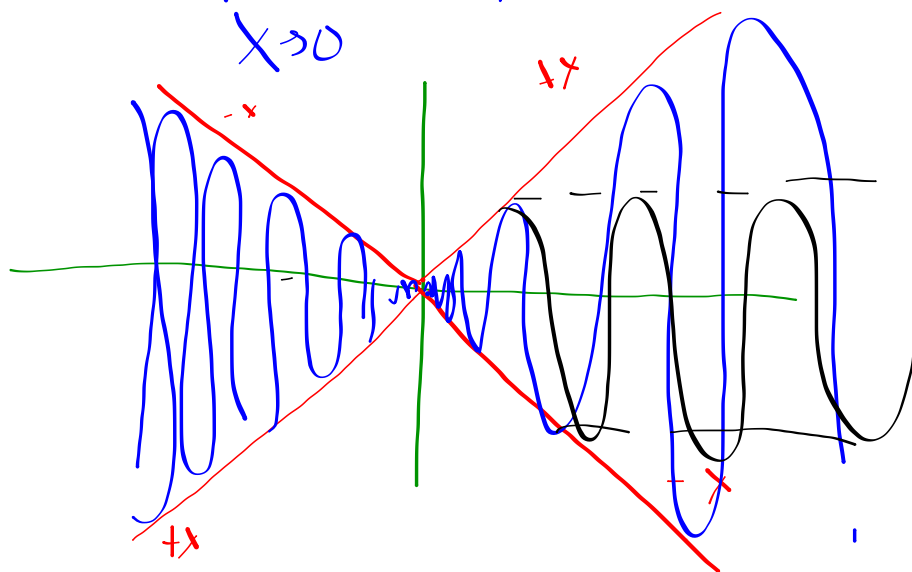
positive

$$-x \leq x \sin \left( \frac{1}{x} \right) \leq x \cdot 1 = x$$

$$\lim_{x \rightarrow 0} -x = 0$$

$$\lim_{x \rightarrow 0} \sin \left( \frac{1}{x} \right) = \text{dne}$$

$$\lim_{x \rightarrow 0} x = 0$$



HW) 2.3

90, 88, 81, 70, 71,

2.4

1-6, 8-9

$$\sin\left(\frac{1}{x}\right) =$$

$$\sin \frac{1}{2} = .479$$

$$2 \sin \frac{1}{2} = .958$$

$$1000 \sin \frac{1}{1000} = .000999 : \times 1000 = .99999$$



2.3 / 47)

$$\lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x - 9}$$

a) substitution?  $\frac{\sqrt{9} - 3}{9 - 9} = \frac{0}{0}$  osh

b) factor & cancel **IDK it**

c) multiply by conjugate

$$\lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x - 9} = \lim_{x \rightarrow 9} \frac{(\sqrt{x} - 3)(\sqrt{x} + 3)}{(x - 9)(\sqrt{x} + 3)} = \frac{(\sqrt{x})^2 - 9}{(x - 9)(\sqrt{x} + 3)}$$

$$= \lim_{x \rightarrow 9} \frac{x - 9}{(x - 9)(\sqrt{x} + 3)} = \lim_{x \rightarrow 9} \frac{1}{\sqrt{x} + 3} = \frac{1}{3 + 3} = \frac{1}{6}$$

substitution